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COMMENT

Comment on 'Approximate analytical solutions of the Dirac equation with the Pöschl–Teller potential including spin-orbit coupling'

H Akcay

Faculty of Engineering, Baskent University, Ankara, Turkey

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Abstract
In this comment we show that the energy eigenvalue equations and the wave functions given by a previous article on the solutions of the Dirac equation with Pöschl–Teller potential must be corrected.

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In a recent paper [1], the Dirac equation is solved for the Pöschl–Teller potential. The paper starts with the following Dirac equation for a fermion with mass M in a scalar potential $S(r)$ and a vector potential $V(r)$:

$$\{\vec{\alpha}\cdot\vec{p}+\beta[M+S(r)]\}\psi=[E-V(r)]\psi \tag{1}$$

and defines the Dirac spinors as

$$\psi_{n\kappa}=\frac{1}{r}\begin{bmatrix}F_{n\kappa}(r)Y^{l}_{jm}(\theta,\phi)\\ iG_{n\kappa}Y^{\tilde{l}}_{jm}(\theta,\phi)\end{bmatrix}, \tag{2}$$

where $F_{n\kappa}(r)$ and $G_{n\kappa}(r)$ are the upper and lower component radial wavefunctions, respectively. Under the condition of exact pseudospin symmetry, they obtain the following equation for the lower component:

$$\left[-\frac{\mathrm{d}^2}{\mathrm{d}r^2}+\frac{\kappa(\kappa-1)}{r^2}-(M-E_{n\kappa}+C)\Delta(r)\right]G_{n\kappa}=\left[E^2_{n\kappa}-M^2-C(M+E_{n\kappa})\right]G_{n\kappa}(r), \tag{3}$$

where $\Delta(r)=V(r)-S(r)$ and $V(r)+S(r)=C$. The difference potential Δ is taken as the Pöschl–Teller potential:

$$\Delta(r)=-\frac{A(A+\alpha)}{\cosh^2(\alpha r)}+\frac{B(B-\alpha)}{\sinh^2(\alpha r)}, \tag{4}$$

and the spin–orbit term is approximated as

$$\frac{\kappa(\kappa-1)}{r^2}\approx\frac{4\alpha^2\kappa(\kappa-1)\,\mathrm{e}^{-2\alpha.r}}{(1-\mathrm{e}^{-2\alpha.r})^2}. \tag{5}$$

Replacing these in equation (3) and defining a new variable $z = -\sinh^2(\alpha r)$, they transform that equation into the following form:

$$z(z-1)\frac{\mathrm{d}^2 G_{n\kappa}(z)}{\mathrm{d}z^2} + \left(\frac{1}{2} - z\right)\frac{\mathrm{d}G_{n\kappa}(z)}{\mathrm{d}z} + \left[-\varepsilon^2 - \frac{\beta}{z} - \frac{\gamma}{1-z}\right]G_{n\kappa}(z) = 0. \tag{6}$$

This equation is correct if the parameters are defined as follows:

$$\varepsilon_{n\kappa}^2 = \frac{E_{n\kappa}^2 - M^2 - C(M + E_{n\kappa})}{4\alpha^2}, \tag{7}$$

$$\beta = -\frac{(M - E_{n\kappa} + C)B(B - \alpha)}{4\alpha^2} + \frac{1}{4}\kappa(\kappa - 1), \tag{8}$$

$$\gamma = -\frac{(M - E_{n\kappa} + C)A(A + \alpha)}{4\alpha^2}. \tag{9}$$

In the paper, the minus signs before the first term in β and in γ are missing. This difference has important consequences. If we take these new parameters and follow their derivations, we find that the energy eigenvalue equation becomes

$$M^2 - E_{n\kappa}^2 + C(M + E_{n\kappa}) = 4\alpha^2\left[-n - \frac{1}{2} + \frac{1}{4}\sqrt{1 - \frac{4(M - E_{n\kappa} + C)A(A + \alpha)}{\alpha^2}} - \frac{1}{4}\sqrt{1 + 4\kappa(\kappa - 1) - \frac{4(M - E_{n\kappa} + C)B(B - \alpha)}{\alpha^2}}\right]^2. \tag{10}$$

The minus signs in front of the last terms inside the square roots are plus signs in the paper. Using their numerical values for the parameters (M, A, B, C,α) in equation (10), it is not possible to find the bound state energy values given in the paper. We also note that the parameters η and δ which are defined in the paper as

$$\eta = \tfrac{1}{4}(1 + \sqrt{1 + 16\beta}), \qquad \delta = \tfrac{1}{4}(1 - \sqrt{1 + 16\gamma})$$

will change according to equations (8) and (9) respectively. The paper gives the wavefunctions in terms of these parameters. The asymptotic behavior of the radial wavefunctions has to be reconsidered with the new parameters.

References

[1] Xu Y, He S and Jia C-S 2008 *J. Phys. A: Math. Theor.* **41** 255302